IMAGING SYSTEMS

PROBLEMS TO BE SOLVED BY STUDENTS (GROUPS OF 2)

Prof. José Manuel Rebordão, V2 (31-1-2020) - 2020/21

Select A or B

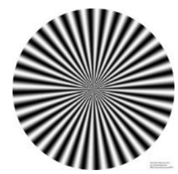
TRACK A – Address only all the problems marked in yellow (until 24:00 h of January 15, 2021)

TRACK B – Select 1 problem in group A and 2 problems in groups B, C and 3 problems in group D (at most. one week following the classes on that topic)

A - SAMPLING

1. Figure 2.2 of the *Theory of Moiré Phenomena, I. Amidror* is the focus of Problems 2.6, 2.7 and 2.8. Address these 3 problems by writing a computer program. Do not hard-code values of angles, amplitudes, offsets, in order to have a flexible tool to analyze the multiplicative superposition of periodic patterns. Ensure that cossinusoidal patterns are well spatially sampled. Use physical units, not arbitrary units, for periods, sampling steps, etc. Try to use the same environment to address also the situation covered in Problem 2.21.

2. Write the analytical model for a radial cossinusoidal pattern, such as the one represented below within a square of side of L. Control the value of the azimuthal angular frequency, by fixing it at L/2 from the center. Now, select a CCD-like square sensor with pitch p, with NxN pixels, and numerically sample your pattern (assuming it is being imaged on the surface of the sensor), plot the sampling matrix, and build its spatial frequencies power spectrum. Increase progressively the value of p and analyze how the image degrades and understand the corresponding changes in the power spectrum, especially at high spatial frequencies.



   Start with L = 1 mm and p = 2 μm. If you have memory problems, reduce the value of L in order to decrease N.

   Plot the image, in order to "see" the impact of sampling, but analyse the spectrum using only 1D profiles through the center, otherwise the information will be difficult to extract.

   Use physical variables, not general "pixel" units and do control the sampling grid and the spatial frequency values.

   Caution: Don't forget that if you use the FFT algorithm, the resolution of the Fourier Transform depends on N; you may need to make zero-padding if you want to generate comparable visual images, although zero-padding is not neutral…

B – DEVICES AND LINEAR SYSTEMS

1. Design (to 1^{st} order) a 170 m focal length Ritchey–Chrétien telescope, coupled to the CCD sensor CCD230-42 Back Illuminated Scientific CCD Sensor, manufactured by E2V. Estimate FOV, mirror diameters and PSF width.

2. Design a microscope of overall magnification of 200 and a field-of-view of 2 mm, using ideal lenses. Ensure that the exit pupil is 1 cm after the ocular. Compute focal lengths, axial distances and diameters of relevant apertures. The system is supposed to be used by a human with normal vision.

3. The properties of focusing and steering devices depend on the required spatial variations on the phase of the incoming wave, both in space or in time. Address the problem 5.3 of *Goodman, Introduction to Fourier Optics (3^{rd} ed, 2005)*.

4. Diffracting structures can act like focusing devices for a wide range of spectrum domains. Analyze the behavior of the *Fresnel Zone Plate*, solving Problem 5.14 of *Goodman, Introduction to Fourier Optics (3^{rd} ed, 2005)*.

5. Solve the problems 5.9, 5.10, 5.11 of *Goodman, Introduction to Fourier Optics (3^{rd} ed, 2005)*.

6. Analyse the performance of the pinhole camera for different assumptions – Problem 6.7 of *Goodman, Introduction to Fourier Optics (3^{rd} ed, 2005).*

7. The *Iterative Fourier Transform Algorithm* (IFTA), also known as *Gerchberg-Saxton* (GS) algorithm, allows to reconstruct an object from its spectrum provided some assumptions can be accepted on its boundary, positiveness, compactness or spectrum phase. It belongs to the category of "retrieval" algorithms, in case waves amplitude and phase are not simultaneously available. It is an important algorithm to compute holograms that generate user-defined images. Read section 6.6.4 of *Goodman* (a lot of information, including Matlab code, can be found on the *internet*) and show that you have implemented, adapted or played with this algorithm.

For example, choose an initial and simple *bounded* image and compute its Fourier transform; discard the spectral phase and do the 'ping-pong' between the direct and reciprocal spaces. The image you selected initially should be partially retrieved… Try to understand if the algorithm is converging, using a suitable metrics. Enjoy!

C - SPATIAL ANALYSIS

1. Represent analytically the amplitude transmission function, *t(x,y)*, of a binary square array (1 cm side) of square apertures (5 μm), pitch of 10 μm in both directions (duty cycle of 50%). Compute its Fourier Transform (FT). [Use all the tools available: δ-Dirac, combs, rect, convolution]. Plot the power spectrum (square of modulus of the FT), and try to see clearly (by adjusting scales and sampling correctly) the effects of the single aperture side, the pitch and side of the envelope aperture.

2. Problem 6.1 of *Goodman, Introduction to Fourier Optics (3rd ed, 2005)* addresses part of the approach to interferometric imaging (the spatial frequency part). Solve it by computing directly the PSF, considering the system is incoherent.

3. Solve the problems 6.10 and 6.12 of Goodman, Introduction to Fourier Optics (3rd ed, 2005).

4. The aperture stop of a typical telescope has the shape of an annular aperture with (usually) straight struts holding the secondary mirror. Assume there are **four** perpendicular struts, aligned in x and y. Model analytically the complete pupil and compute the PSF analytically. Plot the main results in order to identify the effect of the spider' struts and their thickness, *t*, considering the following parameters (taken from ESO VLT): diameters D_1 = 8.2 m, D_2 = 1.1 m; telescope focal length f = 14.4 m; λ = 10 μm. Consider t = 15 mm.

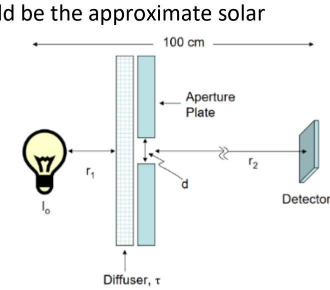
5. Demonstrate you did understand the section *"Review of Coherent Image Formation"*, pages 461-467 of *J.D.Gaskill, Linear Systems, Fourier Transforms and Optics*. You should a) clearly explain the sequence of imaging or filtering models and also b) regenerate the *Example* (numerical application results and Figure 11.6).

D – RADIOMETRY *(Use exclusively SI symbols)*

1. Compute the un-aided human eye retina integrated irradiance of the image of Venus. Use SI units but provide also the result in "number of photons per second, per (μm)^2". Try to take into account the effects of the Earth atmosphere, with suitable (and very well identified) assumptions.

2. Compute the focal plane spectral irradiance of the image of the Sun within the spectral band 550-560 nm, imaged by an ideal lens of 1 m focal length and diameter of 20 cm. Your sensor is the CCD230-42 Back Illuminated Scientific CCD Sensor, manufactured by E2V. Take note of the possibility of saturation, considering that you can play with the diameter of stop aperture.

3. The irradiance from the Sun at the mean radius of the Earths's orbit around the Sun is E_e = 1353 W/m^2 (the *solar constant*). The Sun subtends a diameter of about 0.535 deg at the Earth. Assuming the Sun to be a circular disk, facing the Earth, what is the average radiance L_s of the Sun over the solar disk? If the Sun were a perfect black body at 6000 K, what would be the approximate solar constant?

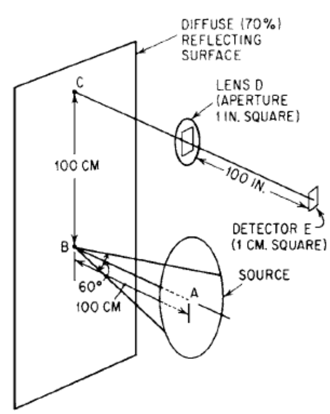


4. A lamp with a radiant intensity of 0.1 W.sr^{-1} illuminates a lambertian diffuser 10 cm away with a 1 cm diameter aperture, located just beyond the diffuser; this secondary 1 cm diameter source then illuminates a detector 100 cm from the lamp. What is the irradiance on the detector if the transmission of the diffuser is 0.60?

5. **A** is a circular source with a radiance of 10 W.sr^{-1}.cm^{-2} radiating uniformly toward plane **BC**. The diameter of **A** subtends 60° from point **B**. The distance **AB** is 100 cm and the distance **BC** is 100 cm.

   An optical system at **D** forms an **image** of the region about point **C** at **E** *(planes through C and E are conjugated)*. Plane **BC** is a diffuse (lambertian) reflector with a reflectivity of 70%. The optical system (**D**) has a 1 in^2 aperture (1 inch = 2.54 cm) and the distance from **D** to **E** is 2.54 m *(100 inches)*. The transmission of the optical system is 80%. We wish to determine the power incident on a 1-cm square photodetector at **E**.

   Determine successively: the irradiance at **B**, the irradiance at **C**, the reflected radiance at **C**, the irradiance of the detector at **E** and finally the power received by the detector.



REFERENCES TO BE USED

I. Amidror, The Theory of the Moiré Phenomenon: Vol. I: Periodic Layers, (2nd ed., Springer-Verlag, 2009)
J. W. Goodman, Introduction to Fourier Optics (3rd ed., Roberts & Company, 2005)
J. D. Gaskill, Linear Systems, Fourier Transforms and Optics (Wiley, 1978)